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(Industrial Management Institute)

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.(K J i) k j i : X_{ijk}
i = , , → (= = =)
J = , , → (= = =)
K = , ,

:

: X

: X

j i : C_{ij}

(j i × ×) - : A_{ij}

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$$\text{Max } Z_0 = \frac{1}{X} + X + X + \frac{1}{X} + \frac{1}{X} + X + \frac{1}{X}$$

$$+ \frac{1}{X} + X + X \quad X \quad X + \frac{1}{X} \quad X - \frac{1}{X}$$

$$\frac{1}{X} - X + X$$

:

$$X \left[\left(\frac{1}{X} \right) \left(\frac{1}{X} \right) + \left(\frac{\text{mg}}{\text{kg}} \right) \times \times \times \left(\frac{\text{kg} \times \text{Kg}}{\text{mg}} \right) \right]$$

$$+ X \left(\frac{1}{X} \right) + X \left(\frac{1}{X} \right) +$$

$$X \text{ kg} \left(\frac{\text{kg}}{\text{kg}} + \frac{\text{mg}}{\text{kg}} / \times \left(\frac{\text{kg} \times \text{kg}}{\text{mg}} \right) + \right.$$

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$$X + X + X + X + X + X <$$

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$$X + X + X + X + X + X <$$

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$$X \times \times \quad \times \quad + X \times \quad \times \quad +$$

$$U \quad U$$

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$$X \text{ Kg} \times \frac{\text{mg}}{\text{Kg}} \times \frac{\text{Kg}}{\text{mg}} \times / +$$

:

$$\frac{1}{X} + \frac{1}{X} + \frac{1}{X} + \frac{1}{X} + \frac{1}{X} + \frac{1}{X} <$$

$$\frac{1}{X} + \frac{1}{X} + \frac{1}{X} + \frac{1}{X} + \frac{1}{X} + \frac{1}{X} <$$

$$\frac{1}{X} + \frac{1}{X} + \frac{1}{X} + \frac{1}{X} + \frac{1}{X} + \frac{1}{X} >$$

$$\frac{1}{X} + \frac{1}{X} + \frac{1}{X} + \frac{1}{X} + \frac{1}{X} + \frac{1}{X} <$$

$$\frac{1}{X} + \frac{1}{X} + \frac{1}{X} + \frac{1}{X} + \frac{1}{X} + \frac{1}{X} >$$

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Excel

WinQsb DS

DS

Linear Programming

	X111	X211	X311	X112	X212	X312	X113	X213
Maximize	132.5	43	718	58.4	45.1	652	31.2	84.2
Constraint 1	1	0	0	1	0	0	1	0
Constraint 2	0	1	0	0	1	0	0	1
Constraint 3	0	0	1	0	0	1	0	0
Constraint 4	1	1	1	0	0	0	0	0
Constraint 5	0	0	0	1	1	1	0	0
Constraint 6	0	0	0	0	0	0	1	1
Constraint 7	0	0	0	0	0	0	0	0
Constraint 8	0	0	0	0	0	0	0	0
Constraint 9	0	0	0	0	0	0	0	0
Constraint 10	0.000475	0	0	0.00056	0	0	0.00034	0
Constraint 11	0	0.00045	0	0	0.00059	0	0	0.00049
Constraint 12	0	0.00045	0	0	0.00059	0	0	0.00049
Constraint 13	0	0	0.00255	0	0	0.0028	0	0
Constraint 14	0	0	0.00255	0	0	0.0028	0	0
Solution->	0	0	30000	0	10000	0	0	3000.002

	X313	X121	X221	X321	X122	X222	X322	X131
Maximize	820	412	-176	-328	197.5	-284	-445.85	-243.6
Constraint 1	0	1	0	0	1	0	0	1
Constraint 2	0	0	1	0	0	1	0	0
Constraint 3	1	0	0	1	0	0	1	0
Constraint 4	0	0	0	0	0	0	0	0
Constraint 5	0	0	0	0	0	0	0	0
Constraint 6	1	0	0	0	0	0	0	0
Constraint 7	0	1	1	1	0	0	0	0
Constraint 8	0	0	0	0	1	1	1	0
Constraint 9	0	0	0	0	0	0	0	1
Constraint 10	0	0.0014	0	0	0.001125	0	0	0.00054
Constraint 11	0	0	0.0004	0	0	0.00036	0	0
Constraint 12	0	0	0.0004	0	0	0.00036	0	0
Constraint 13	0.003	0	0	0.00018	0	0	0.000143	0
Constraint 14	0.003	0	0	0.00018	0	0	0.000143	0
Solution->	12000	7425.001	17575	0	0	10000	0	0


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	X231	X331		RHS	DUAL
Maximize	-340	450			
Constraint 1	0	0	<=	80000	0
Constraint 2	1	0	<=	60000	0
Constraint 3	0	1	<=	100000	0
Constraint 4	0	0	<=	30000	704.825
Constraint 5	0	0	<=	10000	912.4
Constraint 6	0	0	<=	15000	804.5
Constraint 7	0	0	<=	25000	412
Constraint 8	0	0	<=	10000	245.2
Constraint 9	1	1	<=	25000	431.9167
Constraint 10	0	0	<=	40	0
Constraint 11	0.00052	0	<=	60	0
Constraint 12	0.00052	0	>=	18	-1470000
Constraint 13	0	0.0035	<=	200	5166.674
Constraint 14	0	0.0035	>=	12	0
Solution->	0	25000		\$40,459,500.	

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Variable	Value	Reduced Cost	Original Val	Lower Bound	Upper Bound
X111	0	572.325	132.5	-Infinity	704.825
X211	0	0.3249756	43	-Infinity	43.32497
X311	30000	0	718	717.675	Infinity
X112	0	854.0001	58.4	-912.4001	912.4001
X212	10000	0	45.1	-229.7667	Infinity
X312	0	274.8667	652	-926.8667	926.8667
X113	0	773.3	31.2	-804.5	804.5
X213	3000.002	0	84.2	83.81767	99.70002
X313	12000	0	820	804.5	820.3823
X121	7425.001	0	412	359	415.8803
X221	17575	0	-176	-179.8803	-123
X321	0	740.93	-328	-412.93	412.93
X122	0	47.70004	197.5	-245.2	245.2
X222	10000	0	-284	-331.7	Infinity
X322	0	691.7863	-445.85	-245.9363	245.9363
X131	0	675.5167	-243.6	-431.9167	431.9167
X231	0	7.516644	-340	332.4834	-332.4834
X331	25000	0	450	442.4834	Infinity
Constraint	Dual Value	Slack/Surplus	Original Val	Lower Bound	Upper Bound
Constraint 1	0	72575	80000	7425	Infinity
Constraint 2	0	19425	60000	40575	Infinity
Constraint 3	0	33000	100000	67000	Infinity
Constraint 4	704.825	0	30000	26470.59	44117.64
Constraint 5	912.4	0	10000	4966.101	21915.25
Constraint 6	804.5	0	15000	12000	29346.94
Constraint 7	412	0	25000	17575	46146.43
Constraint 8	245.2	0	10000	1749.999	29527.77
Constraint 9	431.9167	0	25000	22428.57	35285.71
Constraint 10	0	29.605	40	10.395	Infinity
Constraint 11	0	42	60	18	Infinity
Constraint 12	-1470000	0	18	10.97	20.97
Constraint 13	5166.674	0	200	164	209
Constraint 14	0	188	12	-200	200

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Solution List

X111	NONBasic	0
X211	NONBasic	0
X311	Basic	30000
X112	NONBasic	0
X212	Basic	10000
X312	NONBasic	0
X113	NONBasic	0
X213	Basic	3000.002
X313	Basic	12000
X121	Basic	7425.001
X221	Basic	17575
X321	NONBasic	0
X122	NONBasic	0
X222	Basic	10000
X322	NONBasic	0
X131	NONBasic	0
X231	NONBasic	0
X331	Basic	25000
slack 1	Basic	72575
slack 2	Basic	19425
slack 3	Basic	33000
slack 4	NONBasic	0
slack 5	NONBasic	0
slack 6	NONBasic	0
slack 7	NONBasic	0
slack 8	NONBasic	0
slack 9	NONBasic	0
slack 10	Basic	29.605
slack 11	Basic	42
surplus 12	NONBasic	0
slack 13	NONBasic	0
surplus 14	Basic	188
Optimal Value (Z)		40459502

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	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	X111	0	132.5	0	-572.325	at bound		704.825
2	X211	0	43	0	-0.325	at bound		43.325
3	X311	30,000.00	718	21,540,000.00	0	basic	717.675	M
4	X112	0	58.4	0	-854	at bound	-912.4	912.4
5	X212	10,000.00	45.1	451,000.00	0	basic	-229.7667	M
6	X312	0	652	0	-274.867	at bound	-926.8667	926.8667
7	X113	0	31.2	0	-773.3	at bound	-804.5	804.5
8	X213	3,000.00	84.2	252,600.20	0	basic	83.8176	99.7
9	X313	12,000.00	820	9,839,998.00	0	basic	804.5	820.3824
10	X121	7,425.00	412	3,059,101.00	0	basic	359	415.8808

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11	X221	17,575.00	-176	-3,093,200.00	0	basic	-179.8808	-123
12	X321	0	-328	0	-740.93	at bound	-412.93	412.93
13	X122	0	197.5	0	-47.7	at bound	-245.2	245.2
14	X222	10,000.00	-284	-2,840,000.00	0	basic	-331.7	M
15	X322	0	-445.85	0	-691.786	at bound	-245.9363	245.9363
16	X131	0	-243.6	0	-675.517	at bound	-431.9167	431.9167
17	X231	0	-340	0	-7.5167	at bound	332.4833	-332.4833
18	X331	25,000.00	450	11,250,000.00	0	basic	442.4833	M

Objective Function (Max.) = 40,459,500.00

	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	C1	7,425.00	<=	80,000.00	72,575.00	0	7,425.00	M
2	C2	40,575.00	<=	60,000.00	19,425.00	0	40,575.00	M
3	C3	67,000.00	<=	100,000.00	33,000.00	0	67,000.00	M
4	C4	30,000.00	<=	30,000.00	0	704.825	26,470.59	44,117.64
5	C5	10,000.00	<=	10,000.00	0	912.4	4,966.10	21,915.25
6	C6	15,000.00	<=	15,000.00	0	804.5	12,000.00	29,346.94
7	C7	25,000.00	<=	25,000.00	0	412	17,575.00	46,146.43
8	C8	10,000.00	<=	10,000.00	0	245.2	1,750.00	29,527.77
9	C9	25,000.00	<=	25,000.00	0	431.9167	22,428.57	35,285.71
10	C10	10.395	<=	40	29.605	0	10.395	M
11	C11	18	<=	60	42	0	18	M
12	C12	18	>=	18	0	-1,470,000.00	10.97	20.97
13	C13	200	<=	200	0	5,166.66	164	209
14	C14	200	>=	12	188	0	-200	200

WinQsb DS Linear Programming

X = / kg (U) = ppm (U) = %

$$/ \text{Kg} \times \frac{\text{Mg}}{\text{Kg}} \times \frac{\text{Kg}}{\text{mg}} \times \% = / \text{Kg}$$

X Kg × (mg/kg) × Kg/ ×

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- X = kg (Th) = ppm (Th) = %
- X = / kg (Th) = ppm (Th) = %
- X = kg (Th) = ppm (Th) = %
- X = kg (Th) = ppm (Th) = %

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$$\begin{aligned} \times / \times \% &= / \text{ kg} \\ / \times / \times \% &= / \text{ kg} \\ \times / \times \% &= / \text{ kg} \\ \times / \times \% &= / \text{ kg} \end{aligned}$$

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- X = Kg (Co) = ppm (Co) = %
- X = Kg (Co) = ppm (Co) = %
- X = Kg (Co) = ppm (Co) = %

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$$\begin{aligned} \times / \times \% &= / \text{ kg} \\ \times / \times \% &= \text{ kg} \\ \times / \times \% &= / \text{ kg} \end{aligned}$$

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$$\begin{aligned} & (/ \times) + (/ \times) + (/ \times) + (/ \times) + (/ \times) \\ & + (/ \times) + (\times) + (/ \times) = \end{aligned}$$

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$$\sum ((\quad) \quad k \quad j \quad I \quad \times \quad)$$

$$(\quad / \quad \times \quad) + (\quad \times \quad) + (\quad / \quad \times \quad) + (\quad \times \quad) + (\quad \times \quad)$$

$$+ (\quad \times \quad) + (\quad \times \quad) + (\quad \times \quad) =$$

$$\sum ((C_{ij}) \quad j \quad i \quad \times (X_{ijk}) \quad k \quad j \quad i \quad)$$

.(Optimal Value(Z)=40459502) DS

Value DS

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.(Slack =)

()

() : (Dual Value)

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() (/ Kg)

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: Upper Bound Lower Bound

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.(Slack =)

Slack -
(Dual Value)

Slack -

Slack

.() () ()

() Reduced Cost

/ X₁₁₁

Reduced Cost

(Lower Bound)

Lower bound